

Solving Absolute Value Equations

Main Ideas

- Evaluate expressions involving absolute values.
- Solve absolute value equations.

New Vocabulary

absolute value
empty set

GET READY for the Lesson

Seismologists use the Richter scale to express the magnitudes of earthquakes. This scale ranges from 1 to 10, with 10 being the highest. The uncertainty in the estimate of a magnitude E is about plus or minus 0.3 unit. This means that an earthquake with a magnitude estimated at 6.1 on the Richter scale might actually have a magnitude as low as 5.8 or as high as 6.4. These extremes can be described by the absolute value equation $|E - 6.1| = 0.3$.



Absolute Value Expressions The **absolute value** of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number x .

KEY CONCEPT

Absolute Value

Words For any real number a , if a is positive or zero, the absolute value of a is a . If a is negative, the absolute value of a is the opposite of a .

Symbols For any real number a , $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$.

When evaluating expressions, absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

EXAMPLE Evaluate an Expression with Absolute Value

1 Evaluate $1.4 + |5y - 7|$ if $y = -3$.

$$\begin{aligned} 1.4 + |5y - 7| &= 1.4 + |5(-3) - 7| \\ &= 1.4 + |-15 - 7| \\ &= 1.4 + |-22| \\ &= 1.4 + 22 \\ &= 23.4 \end{aligned}$$

Replace y with -3 .

Simplify $5(-3)$ first.

Subtract 7 from -15 .

$$|-22| = 22$$

Add.

CHECK Your Progress

1A. Evaluate $|4x + 3| - 3\frac{1}{2}$ if $x = -2$.

1B. Evaluate $1\frac{1}{3} - |2y + 1|$ if $y = -\frac{2}{3}$.

Absolute Value Equations Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers a and b , where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

EXAMPLE Solve an Absolute Value Equation

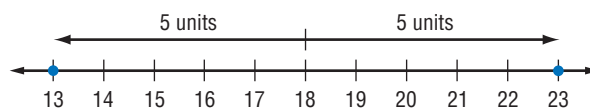
2 Solve $|x - 18| = 5$. Check your solutions.

<p>Case 1</p> $a = b$ $x - 18 = 5$ $x - 18 + 18 = 5 + 18$ $x = 23$	or	<p>Case 2</p> $a = -b$ $x - 18 = -5$ $x - 18 + 18 = -5 + 18$ $x = 13$
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<p>CHECK $x - 18 = 5$</p> $ 23 - 18 \stackrel{?}{=} 5$ $ 5 \stackrel{?}{=} 5$ $5 = 5 \quad \checkmark$	<p>$x - 18 = 5$</p> $ 13 - 18 \stackrel{?}{=} 5$ $ -5 \stackrel{?}{=} 5$ $5 = 5 \quad \checkmark$
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The solutions are 23 and 13. Thus, the solution set is $\{13, 23\}$.

On the number line, we can see that each answer is 5 units away from 18.



CHECK Your Progress

Solve each equation. Check your solutions.

2A. $9 = |x + 12|$ 2B. $8 = |y + 5|$

Study Tip

Symbols

The empty set is symbolized by $\{ \}$ or \emptyset .

Because the absolute value of a number is always positive or zero, an equation like $|x| = -5$ is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**.

EXAMPLE No Solution

3 Solve $|5x - 6| + 9 = 0$.

$$|5x - 6| + 9 = 0 \quad \text{Original equation}$$

$$|5x - 6| = -9 \quad \text{Subtract 9 from each side.}$$

This sentence is *never* true. So the solution set is \emptyset .

CHECK Your Progress

3A. Solve $-2|3a - 2| = 6$. 3B. Solve $|4b + 1| + 8 = 0$.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

EXAMPLE One Solution

1 Solve $|x + 6| = 3x - 2$. Check your solutions.

Case 1	$a = b$	or	Case 2	$a = -b$
	$x + 6 = 3x - 2$			$x + 6 = -(3x - 2)$
	$6 = 2x - 2$			$x + 6 = -3x + 2$
	$8 = 2x$			$4x + 6 = 2$
	$4 = x$			$4x = -4$
				$x = -1$

There appear to be two solutions, 4 and -1 .

CHECK Substitute each value in the original equation.


$ x + 6 = 3x - 2$	$ x + 6 = 3x - 2$
$ 4 + 6 \stackrel{?}{=} 3(4) - 2$	$ -1 + 6 \stackrel{?}{=} 3(-1) - 2$
$ 10 \stackrel{?}{=} 12 - 2$	$ 5 \stackrel{?}{=} -3 - 2$
$10 = 10 \checkmark$	$5 = -5$

Since $5 \neq -5$, the only solution is 4. Thus, the solution set is $\{4\}$.

CHECK Your Progress

Solve each equation. Check your solutions.

4A. $2|x + 1| - x = 3x - 4$ **4B.** $3|2x + 2| - 2x = x + 3$

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CHECK Your Understanding

Example 1
(p. 27)

Evaluate each expression if $a = -4$ and $b = 1.5$.

1. $|a + 12|$ 2. $|-6b|$ 3. $-|a + 21| + 6.2$

Example 2
(p. 28)

FOOD For Exercises 4–6, use the following information.

Most meat thermometers are accurate to within plus or minus 2°F .

4. If a meat thermometer reads 160°F , write an equation to determine the least and greatest possible temperatures of the meat.
5. Solve the equation you wrote in Exercise 4.
6. Ham needs to reach an internal temperature of 160°F to be fully cooked. To what temperature reading should you cook a ham to ensure that the minimum temperature is reached? Explain.

Examples 2–4
(pp. 28–29)

Solve each equation. Check your solutions.

7. $ x + 4 = 17$	8. $ b + 15 = 3$
9. $20 = a - 9 $	10. $34 = y - 2 $
11. $ 2w + 3 + 6 = 2$	12. $ 3n + 2 + 4 = 0$
13. $ c - 2 = 2c - 10$	14. $ h - 5 = 3h - 7$



Exercises

HOMEWORK HELP	
For Exercises	See Examples
15–22	1
23–32	2–4
33–34	2

Evaluate each expression if $a = -5$, $b = 6$, and $c = 2.8$.

15. $|-3a|$ 16. $|-4b|$ 17. $|a + 5|$ 18. $|2 - b|$
 19. $|2b - 15|$ 20. $|4a + 7|$ 21. $-|18 - 5c|$ 22. $-|2c - a|$

Solve each equation. Check your solutions.

23. $|x - 25| = 17$ 24. $|y + 9| = 21$
 25. $33 = |a + 12|$ 26. $11 = |3x + 5|$
 27. $8|w - 7| = 72$ 28. $2|b + 4| = 48$
 29. $0 = |2z - 3|$ 30. $|6c - 1| = 0$
 31. $-12|9x + 1| = 144$ 32. $1 = |5x + 9| + 6$

33. **COFFEE** Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F , plus or minus 5 degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

34. **SURVEYS** Before an election, a company conducts a telephone survey of likely voters. Based on their survey data, the polling company states that an amendment to the state constitution is supported by 59% of the state's residents and that 41% of the state's residents do not approve of the amendment. According to the company, the results of their survey have a margin of error of 3%. Write and solve an equation describing the maximum and minimum percent of the state's residents that support the amendment.

Solve each equation. Check your solutions.

35. $35 = 7|4x - 13|$ 36. $-9 = -3|2n + 5|$
 37. $-6 = |a - 3| - 14$ 38. $3|p - 5| = 2p$
 39. $3|2a + 7| = 3a + 12$ 40. $|3x - 7| - 5 = -3$
 41. $16t = 4|3t + 8|$ 42. $-2m + 3 = |15 + m|$

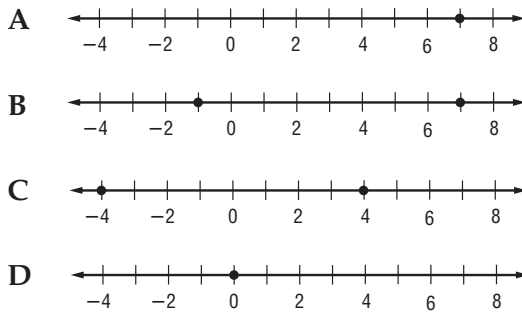
Evaluate each expression if $x = 6$, $y = 2.8$, and $z = -5$.

43. $9 - |-2x + 8|$ 44. $3|z - 10| + |2z|$ 45. $|z - x| - |10y - z|$

46. **MANUFACTURING** A machine fills bags with about 16 ounces of sugar each. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bags the machine will approve.

47. **METEOROLOGY** The *troposphere* is the layer of atmosphere closest to Earth. The average upper boundary of the layer is about 13 kilometers above Earth's surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere.

EXTRA PRACTICE
 See pages 892, 926.
Math online
 Self-Check Quiz at algebra2.com

H.O.T. Problems.**48. OPEN ENDED** Write an absolute value equation and graph the solution set.**CHALLENGE** For Exercises 49–51, determine whether each statement is *sometimes, always, or never* true. Explain your reasoning.**49.** If a and b are real numbers, then $|a + b| = |a| + |b|$.**50.** If a , b , and c are real numbers, then $c|a + b| = |ca + cb|$.**51.** For all real numbers a and b , $a \neq 0$, the equation $|ax + b| = 0$ will have exactly one solution.**52. Writing in Math** Use the information on page 27 to explain how an absolute value equation can describe the magnitude of an earthquake. Include a verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible magnitudes.**STANDARDIZED TEST PRACTICE****53. ACT/SAT** Which graph represents the solution set for $|x - 3| - 4 = 0$?**54. REVIEW** For a party, Lenora bought several pounds of cashews and several pounds of almonds. The cashews cost \$8 per pound, and the almonds cost \$6 per pound. Lenora bought a total of 7 pounds and paid a total of \$48. How many pounds of cashews did she buy?

F 2 pounds

H 4 pounds

G 3 pounds

J 5 pounds

Spiral Review

Solve each equation. Check your solution. (Lesson 1-3)

55. $3x + 6 = 22$

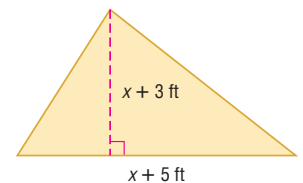
56. $7p - 4 = 3(4 + 5p)$

57. $\frac{5}{7}y - 3 = \frac{3}{7}y + 1$

Name the property illustrated by each equation. (Lesson 1-2)

58. $(5 + 9) + 13 = 13 + (5 + 9)$

59. $m(4 - 3) = m \cdot 4 - m \cdot 3$

GEOMETRY For Exercises 60 and 61, use the following information.The formula for the area A of a triangle is $A = \frac{1}{2}bh$, where b is the measure of the base and h is the measure of the height. (Lesson 1-1)**60.** Write an expression to represent the area of the triangle.**61.** Evaluate the expression you wrote in Exercise 60 for $x = 23$.**GET READY for the Next Lesson****PREREQUISITE SKILL** Solve each equation. (Lesson 1-3)

62. $14y - 3 = 25$

63. $4.2x + 6.4 = 40$

64. $7w + 2 = 3w - 6$

65. $2(a - 1) = 8a - 6$